

2020

Full Marks - 60

Time - 3 hours

The figures in the right-hand margin indicate marks

Answer *all* questions

1. a) What do you mean by generalized coordinates ?
What is the advantage of using them ? 2 + 2
- b) Use Lagrange equation to get the general solution
for a coupled Oscillator. 6
- c) Set up the the Lagrangian and obtain the
Lagrangian equation for a simple pendulum.
Deduce its time period. 2 + 3

OR

- d) Derive Euler Lagrange equation using
calculus of variation and hence obtain the
Lagrange's equations of motion for a system of
particles. Use the definition to find angular
momentum of a particle moving under central
force field. 4 + 4

- e) Use Lagrange's equation of motion for a particle
of mass 'm' falling freely under gravity $g \Rightarrow$
uniform (assumed); Solve the equation to obtain
a relation between distance travelled and time
taken. 5

- f) What is Hamilton's principle ? Explain. 2

2. a) Derive Hamilton's canonical equations of motion
in generalized coordinates. What is the
significance of Hamiltonian ? 4 + 2

- b) Obtain Hamiltonian corresponding to the
Lagrangian $L = ax^2 + by^2 - kxy$. 5

- c) What is a cyclic coordinate ? When a generalised
coordinate is cyclic in Lagrangian, the
corresponding momentum is a constant of
motion; explain. 4

OR

- d) Derive Hamilton's equation of motion and obtain
equation of motion of a charged particle in an
electromagnetic field. 8

- e) The Lagrangian of a particle moving in a plane
under the influence of a central potential is given
by $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{V}{r}$. Obtain generalized
momenta corresponding to coordinates
 r and θ . 4

[3]

- f) Using Hamiltonian find the equation of motion for a simple harmonic oscillator. 3
3. a) State and explain the basic postulates of special theory of relativity. Deduce the formula for addition of velocities in relativistic mechanics. What happens to the relative velocity of light as observed by the observer moving with speed of light. 1 + 3 + 1
- b) Explain what is time dilation. Write briefly on twin paradox. <https://www.odishastudy.com> 3 + 3
- c) Establish mass energy relation $E = mc^2$. Show that photon has zero rest mass. 3 + 1

OR

- d) Show that Lorentz transformation can be regarded as transformation due to a rotation of axes in the 4D Minkowski space. 6
- e) What is relativistic energy? Prove the relation $E^2 - p^2c^2 = m_0^2c^4$. Find the expression for the velocity of a particle in terms of relativistic momentum and energy. 1+3+2

[4]

- f) A clock keeps correct time. With what speed should it be moved relative to an observer so that it may seem to lose 2 minutes in 24 hours. 3
4. a) Show that Lorentz transformations can be regarded as transformations due to rotation of axes in the 4D-Minkowski space. 5
- b) Considering conservation of 4-momentum, determine the relation between mass, momentum and energy. 4
- c) Write notes on Four-D space-Like, time-Like intervals and their significance with a brief account of Light cone. 6

OR

- d) Discuss on conservation of 4-momentum and apply it to decay of an unstable particle. 3 + 4
- e) Discuss Doppler effect from a four-vector perspective. 6
- f) Discuss on four velocity vector. 2