MATH—C-XI

UG 5th Semester Exam., 2021-2022 (R)

Subject : MATHEMATICS

(*Multivariate Calculus*)

Paper—C-XI

Full Marks : 80

Time : 3 hours

Answer all questions

The figures in the right-hand margin indicate marks

Candidates are required to give their answers in their own words as far as practicable

PART – I

1. Answer *all* questions : 1×12

   (*a*) If $z = f(x, y)$, define the first partial derivative of f with respect to x.

   (*b*) Define the gradient $\nabla f(x, y)$.

   (*c*) Evaluate :

$$\lim_{(x,y)\to(1,1)} \frac{xy}{x^2 + y^2}$$

   (*d*) What is a critical point of a function of two variables ?

   (*e*) What is the curl of a constant vector field ?

   (*f*) Find div$R$, if $R = x\hat{i} + y\hat{j} + z\hat{k}$.

   (*g*) If $x = r\cos\theta, y = r\sin\theta$, find the Jacobian $\frac{\partial(x, y)}{\partial(r, \theta)}$.

   (*h*) Write two types of iterated integrals.

   (*i*) Find the area of the region D between $y = \cos x$ and $y = \sin x$ over the interval $0 \le x \le \frac{\pi}{4}$ using single integral.

   (*j*) State Fubini's theorem over a parallelopiped in space.

(*k*) State the divergence theorem in the plane.

(*l*) State subdivision rule in line integral.

PART – II

2. Answer any *eight* questions of the following within *two* to *three* sentences : 2 × 8

(*a*) If $\lim_{(x,y)\to(0,0)} \frac{x^2 + x - xy - y}{x - y} = a + 1$, find a.

(*b*) If $f(x,y) = \cos xy^2$, find f_{xx}.

(*c*) If $f(x,y) = 0$ and $g(y,z) = 0$, show that

$$\frac{\partial f}{\partial y}\frac{\partial g}{\partial z}\cdot\frac{dz}{dx} = \frac{\partial f}{\partial x}\frac{\partial g}{\partial y}.$$

(*d*) Find $\nabla f(x,y)$, where $f(x,y) = \frac{y}{x} + \frac{x}{y}$.

(*e*) Let $R = x\hat{i} + y\hat{j} + z\hat{k}$. Then find the unit vector in the direction of R.

(Turn Over)

(*f*) Find the critical points of the function $f(x,y) = 2x^3 - 4xy + y^2 + 2$

(*g*) Find curl (grad f), where f has continuous second order partial derivatives.

(*h*) Evaluate

$$\int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta.$$

(*i*) Evaluate the line integral

$$\int_C (y\hat{i} + x\hat{j})\cdot dR,$$

where C is any path from (0, 0) to (2, 4) using Fundamental Theorem of line integral.

(*j*) Evaluate

$$\iint_S xy\,dS,$$

when $S: z = 2 - y,\ 0 \le x \le 2,\ 0 \le y \le 2.$

PART – III

3. Answer any *eight* questions of the following within 75 words : 3 × 8

(Continued)

(a) Evaluate

$$\iint_S (x^2 + y^2)\, dS,$$

where S is the surface of the paraboloid $x^2 + y^2 = 2 - z$ above the xy-plane.

(b) Evaluate

$$\iint_S F \cdot N\, dS$$

where $F = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 4$.

(c) Evaluate $\int_C (x\,dy - y\,dx)$ for

$C : 2x - 4y = 1,\ 4 \le x \le 8.$

(d) Evaluate $\iint_R (2x + 3y)\, dx dy$ over

$R : 0 \le x \le 1, 0 \le y \le 2.$

(e) If y is a differentiable function of x such that

$$x \cos y + y \tan^{-1} x = x,$$

then find $\frac{dy}{dx}$, using partial derivative.

(f) Let a function f be defined by

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x^2 + y^6}, & (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Is f continuous at $(0, 0)$?

(g) Find the equation of tangent plane to the surface

$$x^2 + y^2 + z^2 = 3 \text{ at } P_0(1, -1, 1).$$

(h) If A be a constant vector and $R = x\hat{i} + y\hat{j} + z\hat{k}$, show that

$$\text{curl}\,(A \times R) = 2A$$

(i) Let $z = f(x, y)$, where $x = t + \cos t$ and $y = e^t$. If $f_x(1, 1) = 4$ and $f_y(1, 1) = -3$, find $\frac{dz}{dt}$ when $t = 0$.

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(f) If $z = xy\, f\left(\frac{y}{x}\right)$, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z,$$

using Euler's theorem

PART – IV

Answer any four questions choosing (a) or (b) from each of the following 7 × 4

4. (a) If $f(u, v, w)$ is differentiable and $u = x - y$, $v = y - z$ and $w = z - x$, then find

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$$

Or

(b) Show that the following function f is not differentiable at (0, 0) :

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

5. (a) Find the minimum of $x^2 + y^2 + z^2$ subject to the condition

$$ax + by + cz = 1,$$

where $a \neq 0,\ b \neq 0,\ c \neq 0$.

Or

(b) Evaluate

$$\iint_R (x^2 + y^2 + 1)\, dA,$$

where D is the region inside the circle $x^2 + y^2 = 4$.

6. (a) Compute

$$\iint_D \left(\frac{x - y}{x + y}\right)^5 dy\, dx,$$

where D is the region in the xy-plane that is bounded by the coordinate axes and the line $x + y = 1$, using change of variables.

Or

(b) Find the volume of the tetrahedron T bounded by the plane $2x + y + 3z = 6$ and the coordinate planes $x = 0, y = 0$ and $z = 0$.

(Continued)

7. (*a*) Verify Stoke's theorem for the vector function and surface given by $F = y\hat{i} + xz\hat{j} + \hat{k}$ and S is the upper hemisphere

$$x^2 + y^2 + z^2 = 1, z \geq 0$$

Or

(*b*) Verify that the following line integral is independent of path and find its value :

$$\int_{(0,0)}^{(1,1)} \left[\left(3x^2 + 2x + y^2\right)dx + \left(2xy + y^3\right)dy\right].$$