Total Pages—6

**B.Tech-4**

**Math-IV**

**Set-1**

MATHEMATICS-IV

*Full Marks* : 70

*Time* : 3 hours

Answer **Q. No. 1** which is compulsory and any **five** questions out of the rest seven questions

*The figures in the right-hand margin indicate marks*

**1.** Answer *all* parts of this questions : 2 × 10

(*a*) Explain rounding and chopping of a number and the error involved in both the cases.

(*b*) State the condition of convergence of the fixed point iteration method.

(*c*) Prove that the interpolating polynomial for a function $f(x)$ at $x_0, x_1, x_2, \ldots, x_n$ is unique.





(*d*) Evaluate $I = \int_0^1 e^{-x^2}\, dx$, by Trapezoidal rule with $n = 6$.

(*e*) State the condition on $f(x)$ for error estimation and mention the error formula in Simpson's $\frac{1}{3}$ rule of integration.

(*f*) What is a degenerate feasible solution in an LPP ?

(*g*) Prove that $1+\mu^2\sigma^2 = \left(1+\frac{\delta^2}{2}\right)^2$.

(*h*) Five coins are tossed simultaneously. Find the probability of the event $A$ : At least one head turns up. Assume that the coins are fair.

(*i*) State the normal distribution function and plot it.

(*j*) Define random variable.

2. (*a*) Find a positive root of $x^4 - x = 10$ using Newton's method, do only three iterations by suitably choosing a point. 5

(*b*) Find a form $x = g(x)$ of $f(x) = x^2 - 2 = 0$, that gives convergence to the root near 1. Perform 4 iterations to find the root $x$ starting with 1. 5

3. (*a*) Using Newton's forward formula compute the pressure of the steam at temperature $142^\circ$ from the following steam table : 5

| Temperature | 140 | 150 | 160 | 170 | 180 |
|---|---|---|---|---|---|
| Pressure | 3·685 | 4·854 | 6·302 | 8·076 | 10·225 |

(*b*) Integrate $f(x) = \frac{1}{4}\pi x^4 \cos \frac{1}{4}\pi x$, from $x = 0$ to 2 by Simpson's $\frac{1}{3}$rd rule with $n = 5$. 5





4. (*a*) Starting from $x = 1$, $y = 1$, $z = 1$, solve the following system of linear equations by Gauss-Seidel method (do three iterations) : 5

$$5x + y + 2z = 19$$
$$x + 4y - 2z = -2$$
$$2x + 3y + 8z = 39.$$

(*b*) Apply power method starting from $[1, 1, 1]^T$ to compute the eigenvector and eigenvalue of the following matrix, do three iterations.

$$\begin{pmatrix} 3\cdot6 & -1\cdot8 & 1\cdot8 \\ -1\cdot8 & 2\cdot8 & -2\cdot6 \\ 1\cdot8 & -2.6 & 2\cdot8 \end{pmatrix}.$$ 5



5. (*a*) Solve the initial value problem $y' + 5x^4y^2 = 0$, $y(0) = 1$, $h = 0{\cdot}2$ by Eulers method. 5

(*b*) Using Runge-Kutta 4th order method find the solution of $y' = y - x$, $y(0) = 1{\cdot}5$ on $[0, 1]$ by choosing $h = 0{\cdot}2$. 5

6. (*a*) Solve the LPP 5

Maximize $z = 30x_1 + 10x_2$,

subject to $-x_1 + x_2 \le -5$,

$2x_1 + x_2 \ge 10$,

$x_2 \ge 4$, $10x_1 + 15x_2 \le 150$.

(*b*) Using an artificial variable : 5

Minimize $f = 4x_1 + x_2 + 2x_3$

subject to $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$,

and $x_1 + x_2 = x_3 \le 0$, $x_1 + x_2 - x_3 \le 0$.

7. (*a*) Five screws are drawn at random from a lot of 100 screws, 10 of which are defective. Find the probability of the event that all 3 screws drawn are non defective, assuming that we draw (*i*) with replacement (*ii*) without replacement. 5

(*b*) Sketch the density $f(x) = \frac{1}{4}$ $(2 < x < 6)$ and the distribution function. Find $P(X \ge 4)$, $P(X \le 3)$. 5





8. (*a*) Determine a 95 % confidence interval for the mean $\mu$ of a normal population with variance $\sigma^2 = 16$, using a sample of 200 with mean 74·81. 5

(*b*) Compute the correlation coefficient between $X$ and $Y$. 5

| $X =$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| $Y =$ | 123 | 106 | 114 | 128 | 113 | 109 | 120 | 102 | 111 |