

**B.Tech-3rd(All Br.)  
Mathematics-III**

*Full Marks : 70*

*Time : 3 hours*

Answer six questions including Q. No. 1  
which is compulsory

*The figures in the right-hand margin indicate marks*

1. Answer the following questions : 2 × 10

- (a) Express  $\sin(1 - i)$  in  $a + ib$  form.
- (b) Find all values of  $(-1 + i)^{-3i}$ .
- (c) Show that the function  $\bar{z}$  is nowhere differentiable.
- (d) Find  $\oint_C \bar{z} dz$  where  $C$  is the unit circle about the origin.
- (e) Write the maximum modulus theorem.

(f) Obtain the partial differential equation from  $z = axy + b$  by eliminating arbitrary constants  $a$  and  $b$ . http://www.odishastudy.com

(g) Classify the one dimensional wave equation as parabolic or hyperbolic.

(h) Evaluate  $\int_C y^2 dx - 2x^2 dy$  where  $C$  is the parabola  $y = x^2$  from  $(0, 0)$  to  $(2, 4)$ .

(i) Find a unit normal vector of the surface represented by  $x^2 + y^2 + z^2 = 36$ .

(j) Evaluate  $\int_C \frac{dz}{z-1}$   $C : |z| = 1$ .

2. (a) Find all solutions of the equation  $z^4 - (3 + 6i)z^2 - 8 + 6i = 0$ . 5

(b) Determine the set of all  $z$  satisfying the equation  $|z| + \operatorname{Re}(z) = 0$ . 5

3. (a) Find the linear fractional transformation  $w = f(z)$  which maps  $i, 1, 2$  in  $z$ -domain to  $i, -i, 1 + i$  in  $w$ -domain respectively. 5

4. Determine the radius of convergence and open disk of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)} (z+8i)^n. \quad 5$$

5. Evaluate  $\oint_C \frac{z^4}{z-2i} dz$  where  $C$  is any closed curve enclosing  $2i$ . 5

6. Evaluate  $\oint_C \frac{z \sin(3z)}{(z+4)^3} dz$ , where  $C$  is the circle  $|z-2i|=9$ . 5

7. Evaluate  $\oint_C \frac{z}{\sinh^2(z)} dz$ , where  $C$  is the circle of radius 1 about  $\frac{1}{2}$ . 5

- (b) Suppose  $f$  is differentiable in an open disk about zero and satisfies  $f''(z) = 2f(z) + 1$ . Suppose  $f(0) = 1$  and  $f'(0) = i$ . Find the Maclaurin expansion of  $f(z)$ . 5

6. Find the temperature at any arbitrary time in a bar of length 2 whose ends are kept at zero and lateral surface insulated if the initial temperature is  $\sin \frac{\pi x}{2} + 3 \sin \frac{5\pi x}{2}$ . 10

7. Find the deflection of a string stretched between two fixed points at a distance  $2c$  apart when the string is initially at rest and have initial velocity  $v(x)$ , where

$$v(x) = \begin{cases} \frac{x}{c}, & 0 < x < c \\ \frac{2c-x}{c}, & c < x < 2c \end{cases} \quad 10$$

8. (a) Using residues evaluate  $\int_0^{2\pi} \frac{d\theta}{3+\sin\theta}$ . 5

- (b) Evaluate  $\int_{-\infty}^{\infty} \frac{dn}{(n^2+1)^2}$ . 5