

Full Marks : 70

Time : 3 hours

Answer Q. No. 1 which is compulsory and any five questions from the rest.

*The figures in the right-hand margin indicate marks.*

1. Answer all questions:

[2 x 10]

(i)  $F(n) = n^3 + n \log n$ , then find  $O(F(n))$ .

(ii)  $f(n) = n^3$ ,  $g(n) = n^2$ , then find  $O(fg(n))$ .

(iii)  $T(n) = 7T(n/3) + n^2$ , then find  $O(T(n))$ .

(iv)  $T(n) = 2T(n/4) + \sqrt{n}$ , then find  $O(T(n))$ .

(v) What is graph coloring problem? Give an example.

(vi) What is a flow network? What is skew symmetry property?

(vii) Construct a max heap tree with the data {C, O, M, P, U, T, E, R}.

(viii) What is memoization? How is it different from dynamic programming?

(ix) What is the basic difference between NP-hard and NP-complete problems?

(x) What do you mean by the convex hull of a given set of points? Give a suitable example.

2. (a) Differentiate between aggregate analysis and potential method of amortized analysis with suitable examples. [5]

(b) What are randomized algorithms? Explain the use of indicator random variables in randomized algorithms with a suitable example. [5]

3. (a) Explain the divide-and-conquer technique of analyzing algorithms taking merge sort as an example and derive its worst-case running time. [5]

(b) What are the elements of dynamic programming? Determine a longest common subsequence of  $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$  and  $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$  using dynamic programming and show the table entries. [5]

4. (a) Find an optimal solution to the Knapsack instance  $n = 7$ ,  $m = 24$ ,  
 $w = \{ 2, 3, 5, 7, 1, 4, 6 \}$  and  $P = \{ 10, 5, 15, 7, 6, 18, 3 \}$ . [5]
- (b) Explain why 0 / 1 Knapsack problem cannot be solved optimally using greedy method with a suitable example. [5]
5. (a) Explain the backtracking technique of analyzing algorithms with a suitable example. [5]
- (b) Discuss Prim's algorithm to find the spanning tree of a given graph with a suitable example. [5]
6. (a) What is the basic difference between P, NP and NP-complete problems ? Prove that the circuit-satisfiability problem belongs to the class NP. [5]
- (b) Explain the Rabin-Karp string matching algorithm with a suitable example. [5]
7. (a) Explain how the vertex-cover problem can be solved using an approximation algorithm. [5]
- (b) Discuss Graham's algorithm to find the convex hull of a given set of points. [5]
8. (a) Explain TVSP problem with a suitable example. [5]
- (b) Run the Floyd-Warshall algorithm on the following weighted, directed graph and find the shortest path between each pair of vertices. Show the matrix  $D^{(k)}$  that results from each iteration. [5]